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GREEKS

Delta, Gamma, Vega, Theta, Rho

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Black-Scholes- Merton Model

HYPOTHESIS:

- Stock price follows the geometric brownian motion
- Short selling is allowed
- No transaction and taxation associated costs
- No free risk arbitrage opportunities
- Stock price doesn’t pay dividends
- Interest rates and volatility are constant

FORMULAS

$$Call = S * N(d1) - Ke^{-rT} * N(d2)$$
$$Put = Ke^{-rT} * N(-d2) - S * N(-d1)$$

Where $d1 = (\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right) * \mathrm{T})/\sigma\sqrt{T}$ e d2=1-d1

N(x)= is referred to a variable X distributed as follows: 0 average and 1 standard deviation

GREEKS

DEFINITIONS

-Delta: sensitivity of the option price in respect to the underlying D'(S) Black-Scholes

-Gamma: acceleration rate of the Delta D''(S) Black-Scholes

-Theta: sensitivity of the option price in respect to the time (es days..)

-Rho: sensitivity of the option price in respect to interest rates movements

-Vega: sensitivity of the option price in respect to the volatility

PURPOSES

risk hedging/ speculation

DELTA

-It is the first derivative of a call/put option in respect to the underlinying, respectively DC/DS e DP/DS

Short formula : Call=N(d1)
Put=N(d1)-1

Delta positive for Long Call, short Put

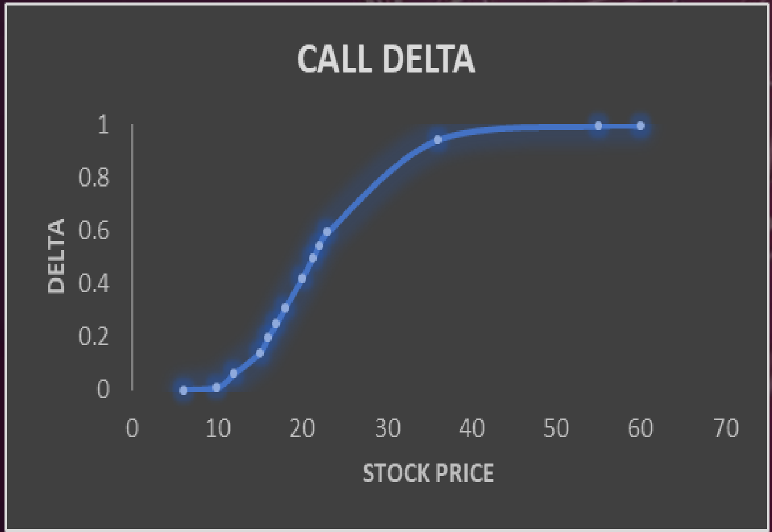
Delta negative for short Call, long Put

It is simply the excerise probability of an option
0<Delta<1 for Call
-1<Delta<0 for Put

DELTA AND UNDERLINYING

call market	call delta	strike	put market	put delta
4.8-5	0.81	17.5	0.9-1	-0.19
3.3-3.5	0.66	20	1.9-2	-0.34
2.35-2.4	0.49	22.5	3.2-3.4	-0.51
1.55-1.6	0.34	25	5-5.1	-0.67
0.7-0.8	0.14	30	9.1-9.3	-0.89

Given the stock price equal to 23, it follows the delta evolution in respect to the strike price changes.
When the option is OTM or deep ITM delta inclination is low.
When the option is ATM delta inclination is at its most.
Position at the money: S=K for call and for Put
Position in the money: S>K or call and viceversa for Put
Position out of the money: S<K for call and viceversa for Put



DELTA AND TIME TO MATURITY

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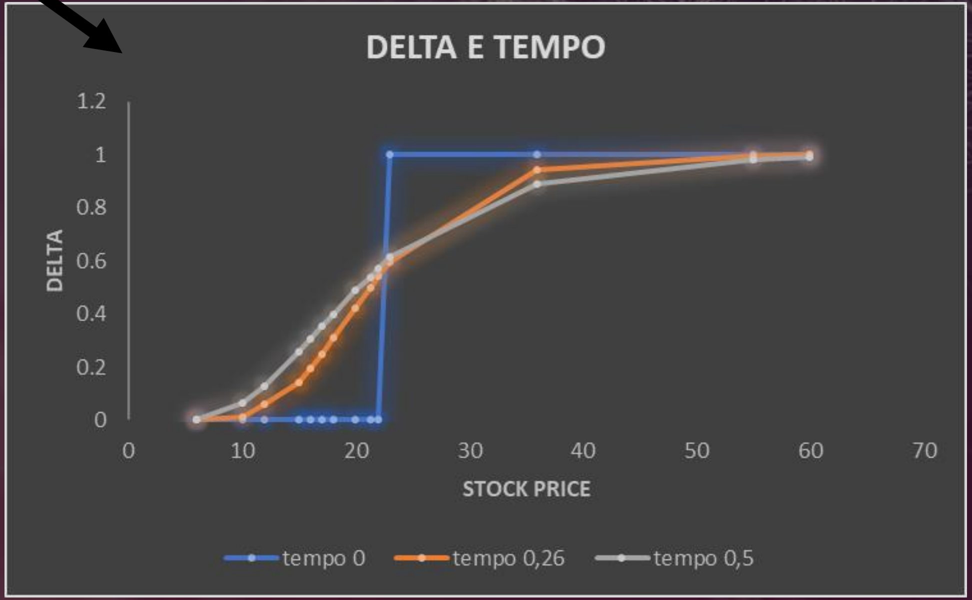
T	0	T	0.26	T	0.5
delta	stock price	delta	stock price	delta	stock price
0	6	*0*	6	*0.004*	6
0	10	*0.011*	10	*0.064*	10
0	12	*0.062*	12	*0.13*	12
0	15	*0.14*	15	*0.26*	15
0	16	*0.197*	16	*0.308*	16
0	17	*0.25*	17	*0.356*	17
0	18	*0.309*	18	*0.4*	18
0	20	*0.42*	20	*0.49*	20
0	21.3	*0.5*	21.3	*0.54*	21.3
0	22	*0.5423*	22	*0.57*	22
1	23	*0.5948*	23	*0.613*	23
1	36	*0.9439*	36	*0.89*	36
1	55	*0.997*	55	*0.98*	55
1	60	*0.999*	60	*0.991*	60
STRIKE 22,5					



Se T ↑ : ITM ↓ e OTM ↑

ATM ↑

This relationship implies that as the time to maturity increases, the time value of the option increases but at the same time the uncertainty of the value that the price of the underlying will assume in the future.

The delta for out of the money and at the money options increases as the market gives a probability that the underlying may vary more in that range and be higher than the strike at maturity. The delta of deep in the money options decreases.

DELTA HEDGING

Absence of arbitrage is assumed according to the B&S model, i.e. the portfolio G=-C+n*S (where C=short call, S=long stock, n=quantity) is risk-free at a

The next instant of time, i.e.

Delta(G)=-Delta(first Call derivative)+n(first derivative S)=0

-Example: Short 1000 Call: K=14, S=13.4, SIGMA=0.2 Rf=5%, t=0.5
Find Delta option and n stocks such that the portfolio is in delta hedging?
N=470

DYNAMIC DELTA HEDGING

Understood as Portfolio rebalancing, i.e. if the option delta changes, the trader makes trades buying or selling instruments(derivative or share)

-Example 2: share price changes to 13.7 with t=0.4, the delta Call=-0.51

Delta portfolio=-0.51*1000+470 other than ZERO

therefore

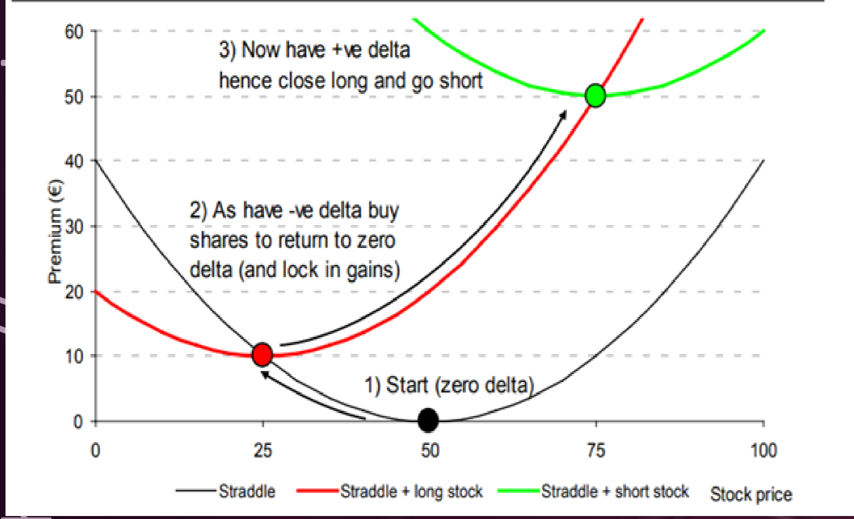
Delta portfolio=0=-0.51*1000+n where n=510, the trader buys 40 more stocks to return to Delta hedging

DELTA HEDGING STRATEGIES

Neutralization of the S-effect on the f derivative

-Dynamic hedging

Figure 27. Locking in Gains through Delta Hedging



1)Short term volatility strategies

C(1)=Co+Gamma+Vega assuming non-constant vola

-Short straddle

-Long straddle

1)SHORT TERM VOLATILITY STRATEGIES: LONG STRADDLE

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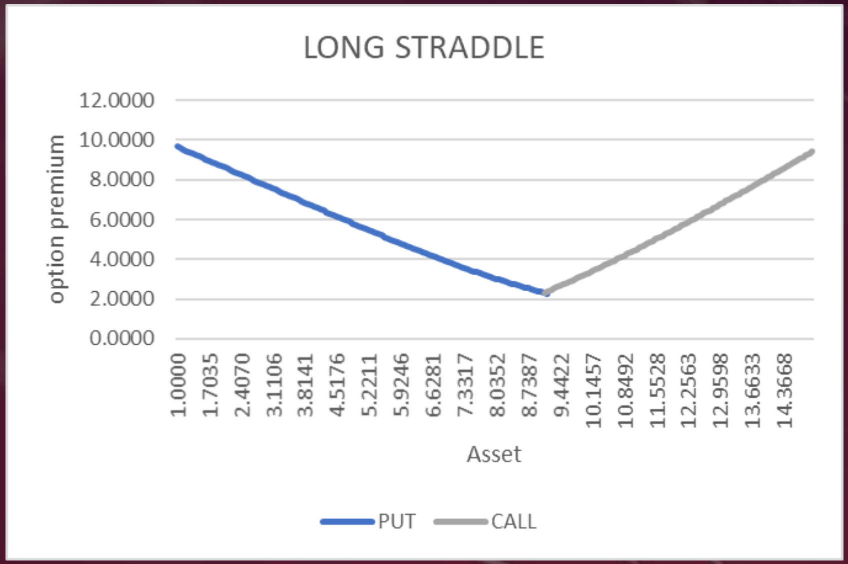
-It is a strategy of speculative purpose: the trader 'bets' on increased volatility and for this reason assumes an initial delta hedging position. It is adapted to short-term trades as the delta hedge has a limited duration and the gain from increased volatility must not be eroded by a reduction in the time interval.

EXAMPLE: The trader bets on increased volatility as the company's balance sheet data will come out, he buys a straddle: BUY PUT AND CALL WITH EQUAL STRIKE. The short straddle is symmetrical and benefits from decreases in volatility.

	PRIMA	DOPO
S	10	10
Vola	50%	80%
Rf	2.50%	2.50%
Life	0.5	0.5
Strike	10.8	10.8
call	1.14	1.98
put	1.8	2.64
delta call	0.498	0.56
delta put	-0.502	-0.43
NET POSITION		
1.68		



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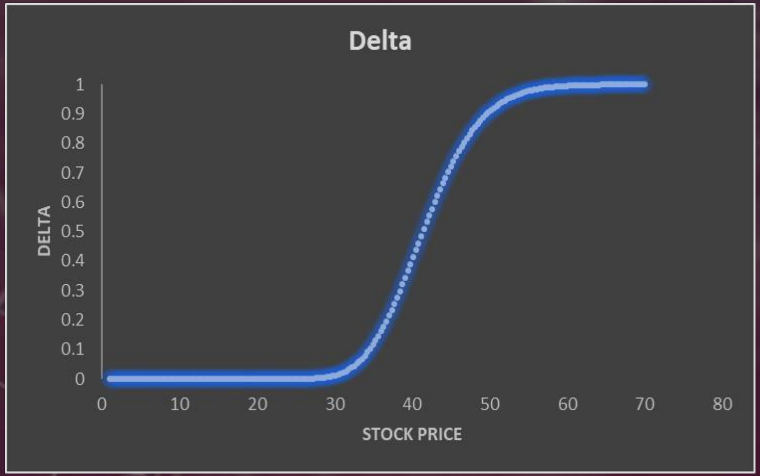
GAMMA: DELTA AND UNDERLYING

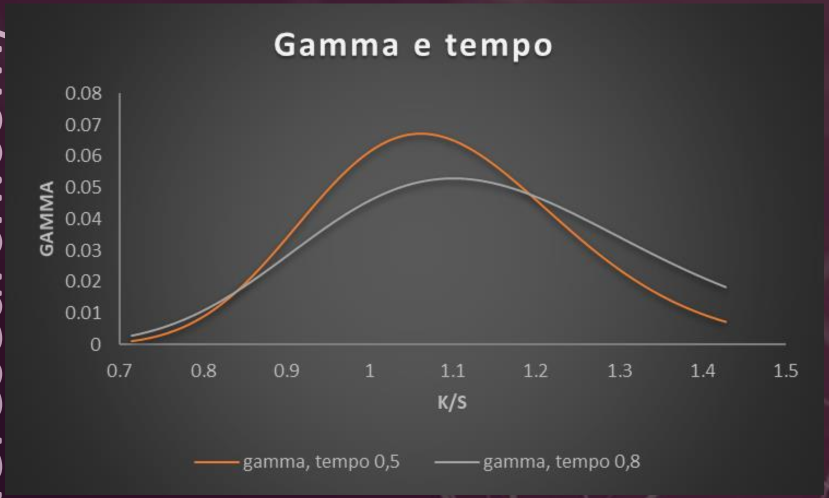
STOCK PRICE	42	43	
DELTA	0.537	0.6	0.537+0.066
GAMMA	0.066	0.063	PENDENZA DIMINUISCE
VOLATILITY 20%			
RISK FREE RATE 10%			

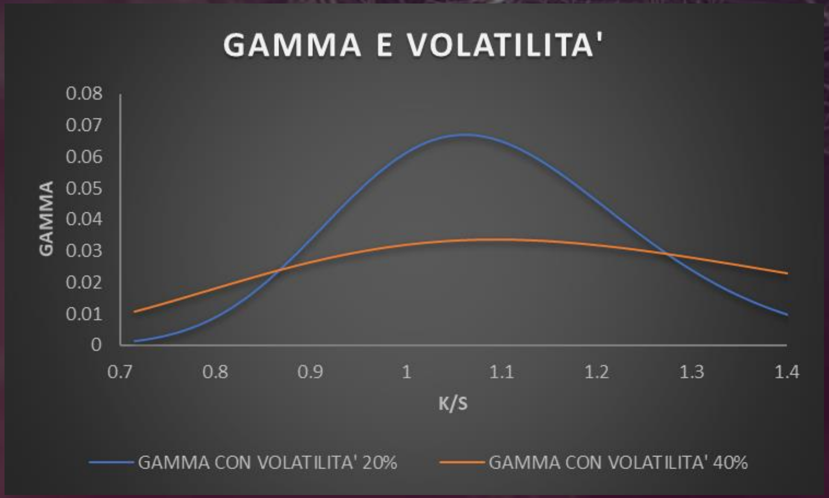


-Since Gamma is the second derivative, it is positive for put and call options. Explain variations of the
Delta as one unit of the underlying varies
-ITM AND OTM low range, ATM high range: depends on slope DELTA

-LONG GAMMA = LONG PUT OR LONG CALL, positive relationship with buying positions
-SHORT GAMMA= SHORT PUT OR SHORT CALL, negative relationship with selling positions

GAMMA: TIME AND VOLATILITY





	Strike Price	Gamma		Strike Price	Gamma
T 0,8	30	0.002857	T 0,5	30	0.00132
	36	0.019974		36	0.021341
	42	0.045978		42	0.061385
	48	0.051943		48	0.058673
	54	0.036425		54	0.0269
	60	0.018364		60	0.007439

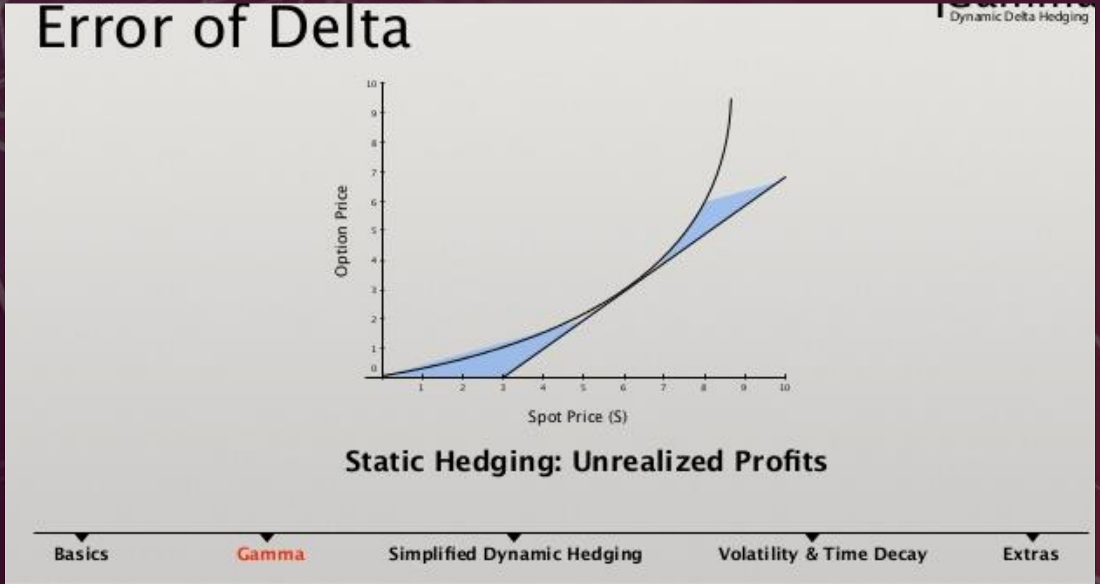
	Strike Price	Gamma		Strike Price	Gamma
VOLA 20%	30	0.00132	VOLA 40%	30	0.010775
	36	0.021341		36	0.023137
	42	0.061385		42	0.031925
	48	0.058673		48	0.033187
	54	0.0269		54	0.028542
	60	0.007439		60	0.021532

GAMMA DELTA HEDGING

It is more accurate than Delta hedging as it captures variations of the process estimated by gamma, in fact:

Call(instant t+1)=Call(t0)+Delta*dS(stock change)+1/2*Gamma*dS^2 (I don't take theta into account)

Without Gamma the price of the derivative would show an error in pricing not grasping the convexity, in fact:



INPUT

So=14 K=14.5 Rf=0.02 T=0.5 Call(t=0)=0.2469 Delta=0.37 Gamma=0.382

During the day the price increases by 3%, quantify delta effect and gamma effect

Call=C(t0)+ Delta*dS + 1/2Gamma*dS^2

where dS=0.03*14=0.42
dS^2=0.1764

DELTA EFFECT=0.1554
EFFETTO GAMMA EFFECT=0.0336
CALL=0.435

<u>CONCLUSIONS</u>: in an initial delta hedging situation (short call and long stock) the trader would have been Uncovered for a sum equal to the gamma effect, gamma coverage is carried out through the purchase or the sale of an additional option
Follows…..............

TRADING IN GAMMA HEDGE

The trader now wants to include a call derivative in the portfolio to neutralize the gamma effect:

-Cancel derivative'': -Gamma(option sold)+b*Gamma(option to insert)=0

-Found the term b, the new number of shares to buy or sell is found: -Delta1+ n(shares) + b*delta=0

-I reach a final position of delta gamma hedging

ATTENTION: in order to neutralize the impact of delta and gamma it is necessary to respect the order, i.e. Initially neutralize the gamma portfolio and only then neutralize the delta portfolio.

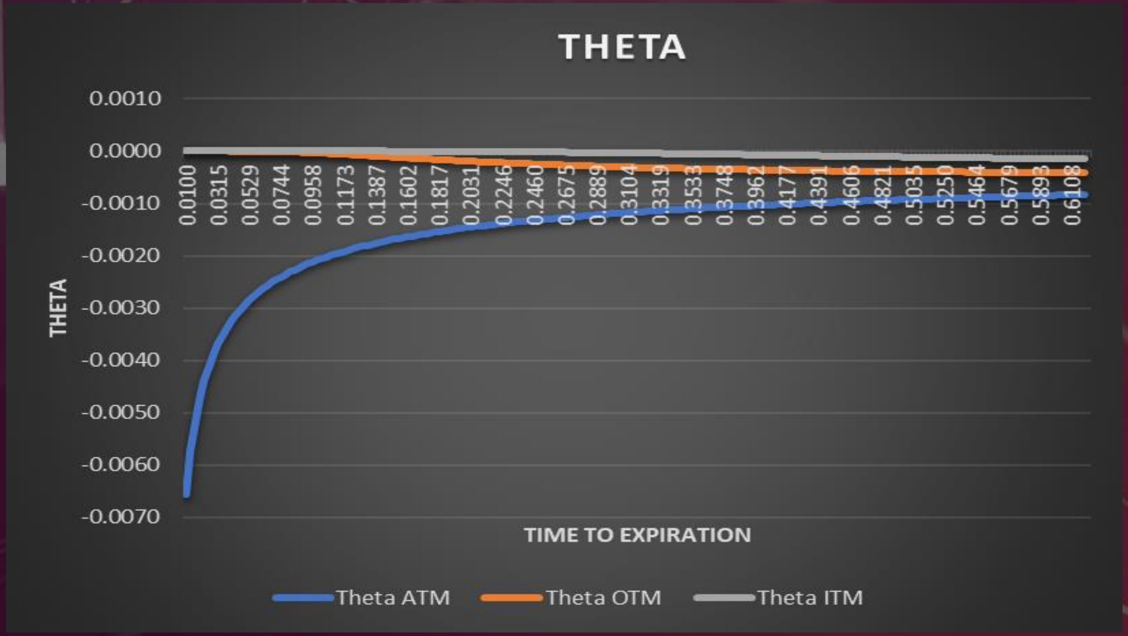
THETA

- It is the derivative of the call or put portfolio with respect to the time θ(c)=θ(fc)/θ(t) where c is the derivative(call), equal for put

- It is a measure of price sensitivity → explains the change in the price of the call / put as time changes

- It is always less than 0 because at the end of the remaining life of the contract, the option tends to be worth less

- It is an annualized value, which therefore must be converted at a daily level



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THETA AND OPTION PRICE

Consider the parameters of the following Call option:

1) If T0=0.5

- S=10
- Volatility=15%
- Risk Free Rate=0 → OPTION PREMIUM= 2.00591
- K= 8
- Theta= -0.0285/254= -0.0001125 daily value

2) If t=0.496, i.e. 1 day has passed and the rest of the parameters remain unchanged then:

C(1)=C(0)+Theta → 2.0058=2.00591-0.0001125

TRADING THETA

The relationship between the position of the treated instrument and time is negative

↓

POSITION	GREEK'S EXPOSURE
LONG CALL	NEGATIVE
LONG PUT	NEGATIVE
SHORT CALL	POSITIVE
SHORT PUT	POSITIVE

HOW TO MAKE PROFIT? SELLING THE PREMIUM!

↓

COVERED CALL

COVERED CALL

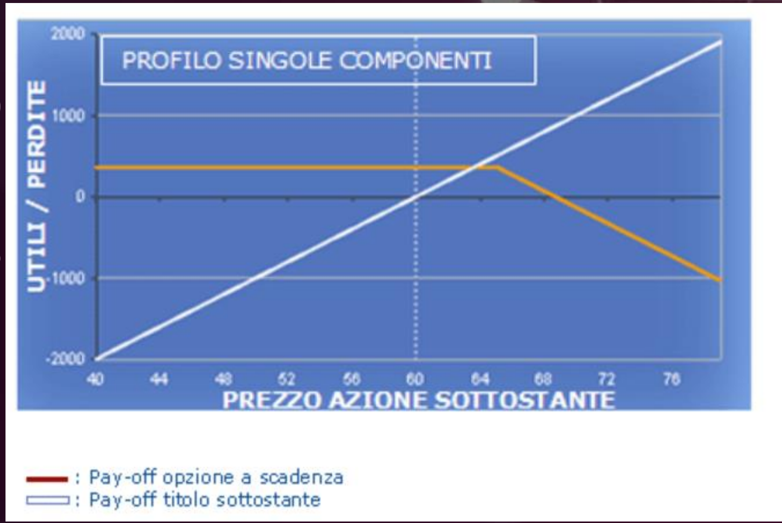
DEFINITION:

- It is a strategy that involves holding a security in the portfolio and selling an otm call

FINALITY

- It has hedging purposes and allows the trader to hedge against any price falls
- Puts a cap on the pay-off at maturity
- The number of options bought must be equal to the number of shares in the portfolio
- Take advantage of the increase in Theta as the deadline approaches, typically 1m expiry.

PAY-OFF



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RHO

DEFINITION

Represents the price sensitivity of the option when rates change by 1%

$$\theta = \frac{\partial C}{\partial r}$$ where C is the call, same formula for Put

$$d(c) = \theta * d(rf)$$ where rf is the risk free rate (Treasury o Swap rate)

POSITION	GREEK'S EXPOSURE
LONG CALL	POSITIVE
LONG PUT	NEGATIVE
SHORT CALL	NEGATIVE
SHORT PUT	POSITIVE

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RHO AND OPTION PRICE

Data:

K=50
T=0.38
S=49
Rf=0.05
vola=0.2
Rho=8,907

How does the value of the call option and the put option change when the rates vary by 0.5%? How are the outstanding positions in the portfolio impacted: long and short?

D(CALL)=8,907* 0.005=0.044
D(PUT)=-0.044

POSITION	GREEK'S EXPOSURE
LONG CALL	0.044
LONG PUT	-0.044
SHORT CALL	-0.044
SHORT PUT	0.044

VEGA

DEFINITION

- It is a measure of sensitivity to the implied volatility of the option
- It describes the price movement of the option as the implied volatility varies by 1pt and is equal to $V=\partial c/\partial \sigma$

FINALITY

- Exposure monitoring
- Volatility trading
- Hedging (rare)

N.B.

- Black-Scholes model assumes constant volatility, the vega in the model should not explain the price changes being equal to 0. In reality, implied volatility is not constant

VEGA RELATIONS

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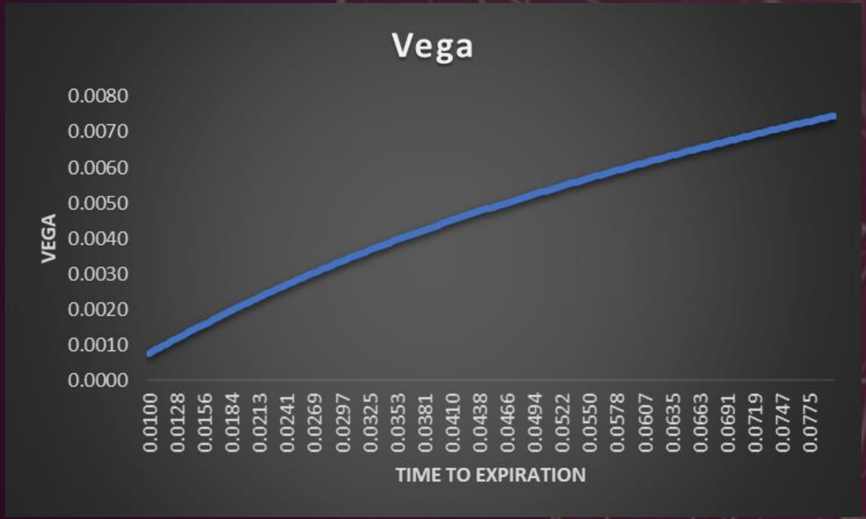
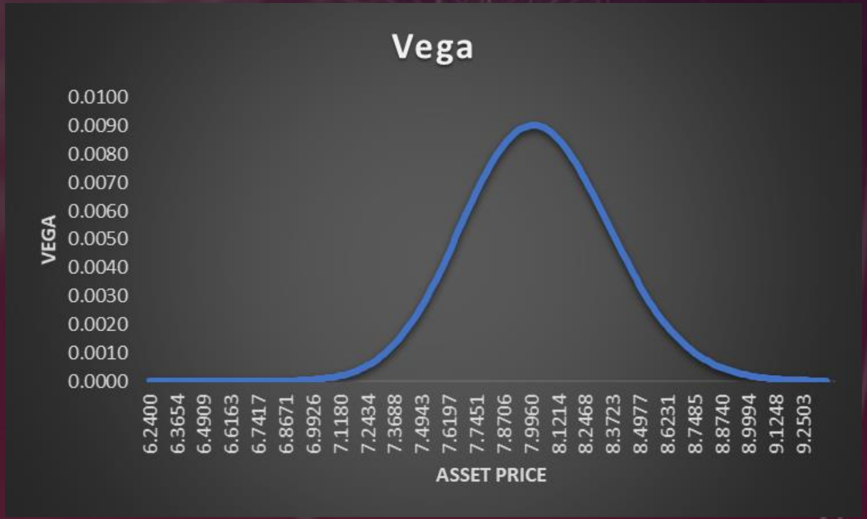
Vega-time to expiration



The value of the vega decreases as the deadline approaches. Option becomes less sensitive to increases in volatility of the underlying close to expiry

Vega-Asset price



The vega is maximum when the option is at the money, vice versa decreases when option is deep in the money and deep out of the money

VEGA AND OPTION PRICE

Data:

K=50
T=0.38
S=49
Rf=0.05
vola=0.2
Vega=12

How does the value of the call option and the put option vary by 1% of the implied volatility? How are the outstanding positions in the portfolio impacted: long and short?

D(CALL)=12* 0.01=0.012
D(PUT)=0.012

POSITION	GREEK'S EXPOSURE
LONG CALL	POSITIVE
LONG PUT	POSITIVE
SHORT CALL	NEGATIVE
SHORT PUT	NEGATIVE

VEGA TRADING

Vega exposure is used for short-term strategies, position liquidated before expiry

STRANGLE

It is a low-cost strategy and takes advantage of the volatility increases of an asset

LONG STRANGLE: LONG CALL E PUT OTM

Long vega

SHORT STRANGLE: SHORT CALL E PUT OTM

Short vega

NB: STRANGLE AND STRADDLE They both represent volatility strategies, however the strangle (long or short) has a lower overall premium as OTM options pay a lower premium

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MANAGING VEGA DURING A LONG STRANGLE

ASSUMPTIONS:
- Trading Vega involves the creation of a short-term structure. It is not brought to maturity
- The trade is executed in Delta hedge

EXAMPLE
-Company A will publish earnings today and the trader wants to take advantage of an increase in volatility by buying a strangle for the reasons listed in the previous slides.

DATI

S=15
K(CALL)=16.5
K(PUT)=13.5
Rf=2%
Vola=20%
Time to expiration=1m

CALL	Q=1
price	0.019
delta	0.054
gamma	0.128
vega	0.005

PUT	Q=1
price	0.010
delta	-0.029
gamma	0.077
vega	0.003

FOLLOWS

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LONG STRANGLE

T=0.38
Volatility=20%

Step 1

- Trader buys 100call OTM

Step 2

- The trader defines the amount of otm put options to buy in order to be in delta hedge using the following formula : $100 * Delta(call = 0.054) + x * Delta(put = 0.029) = 0$ $x = 187$

CALL	Q=100
price	192.260
delta	540.363
gamma	1276.934
vega	47.119

PUT	Q=187
price	182.644
delta	-540.758
gamma	1436.614
vega	53.011

LONG CALL-LONG PUT	EXPOSURE
debit	374.904
delta exposure	-0.395
gamma exposure	2713.548
vega exposure	100.130

LONG STRANGLE

T=0.38
Volatility=30%

How does Greek exposure change? We expect an increase in Vega exposure

CALL	Q=100
price	954.972
delta	1484.391
gamma	1790.258
vega	99.816

PUT	Q=187
price	1180.556
delta	-1861.135
gamma	2528.741
vega	140.990

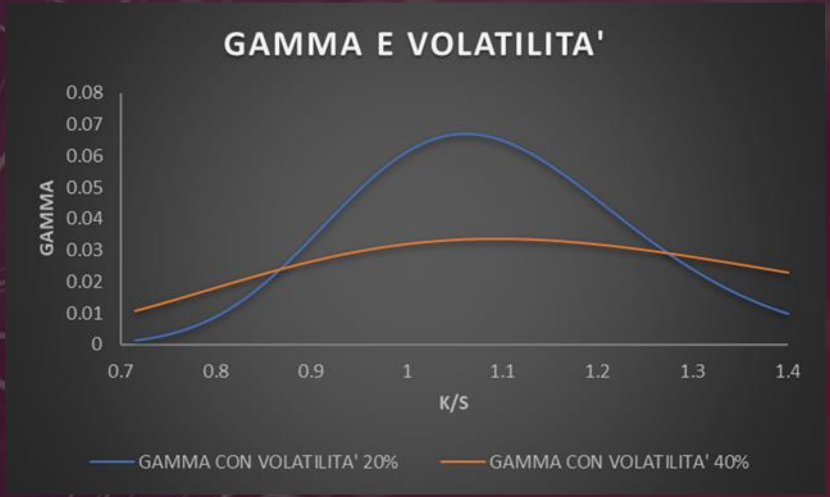
LONG CALL-LONG PUT	EXPOSURE
MARKET VALUE OF THE STRUCTURE	2135.528
delta exposure	-376.744
gamma exposure	4318.999
vega exposure	240.806

NET PROFIT= MARKET VALUE OF THE STRUCTURE-DEBIT= 2135.52-374.9= 1760

CONSIDERATIONS (1)

- Increases the positivity of Vega exposure, as it increases the implied volatility of the underlying

- Increases the Gamma of the structure. It is good to remember that although there have been no price movements of the underlying as volatility increases, the distribution of the Gamma becomes flat. In fact, being both options otm, as volatility increases, the otm and itm range increases, while the atm gamma decreases.



- Increases the delta of positions as increasing implied volatility both options have a higher probability of reaching the strike prices identified by the structure

- The final delta exposure is negative, so I am short the underlying. The return in delta hedge is executed by buying 376 shares of the underlying

CONSIDERATIONS (2)

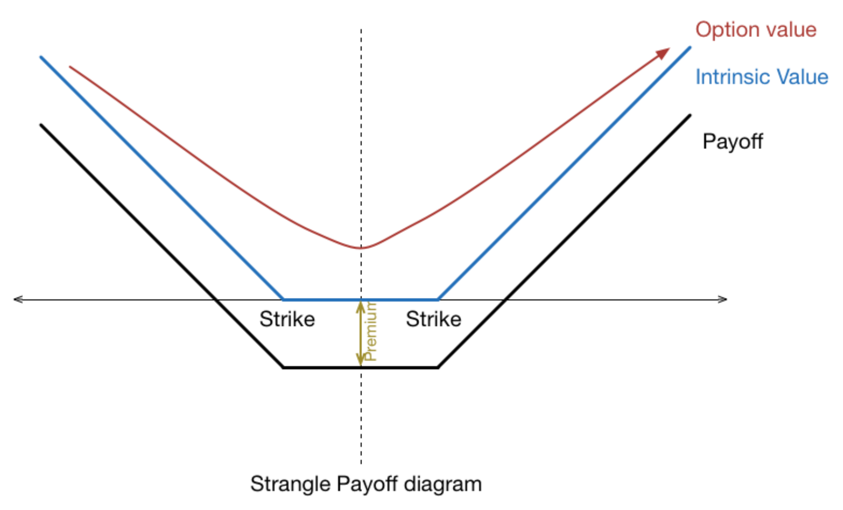
What happens if the structure is kept until it expires?

The payoff of the positions will no longer depend on the Greek but on the pay-off of the individual positions

The initial cost of the structure is determined by the sum of the prizes. COST=CALL+PUT

If at maturity call is in the money then break-even is determined as follows: S-K(1)-COST>0

If at maturity put is in the money then break-even is determined as follows: K(2)-S-COST>0



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MODEL VS REALITY

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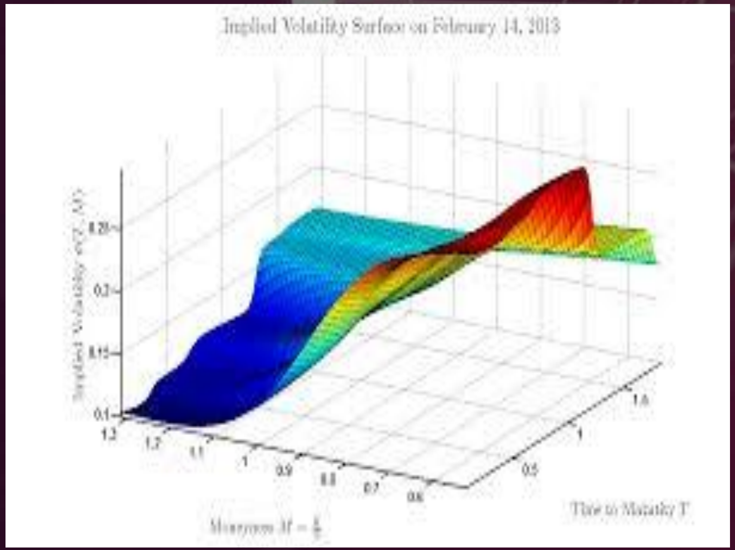
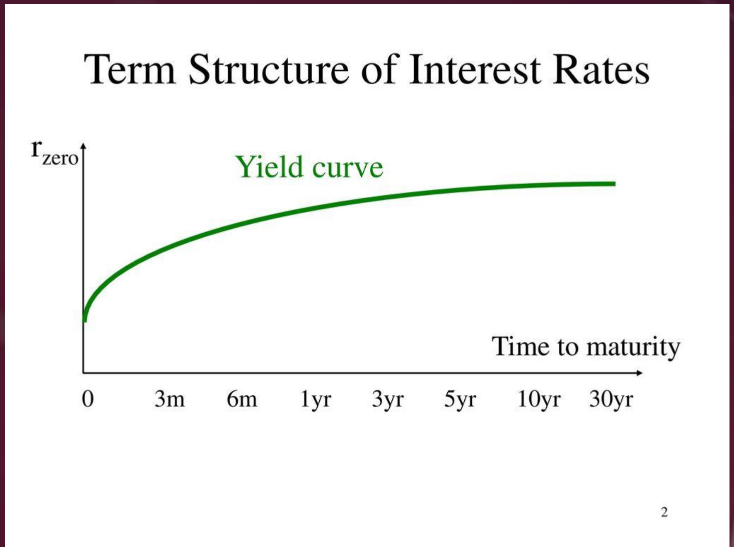
Black Scholes Model assumptions:

- Constant volatility
- Rf constant
- No transaction and taxations costs
- No arbitrage opportunity

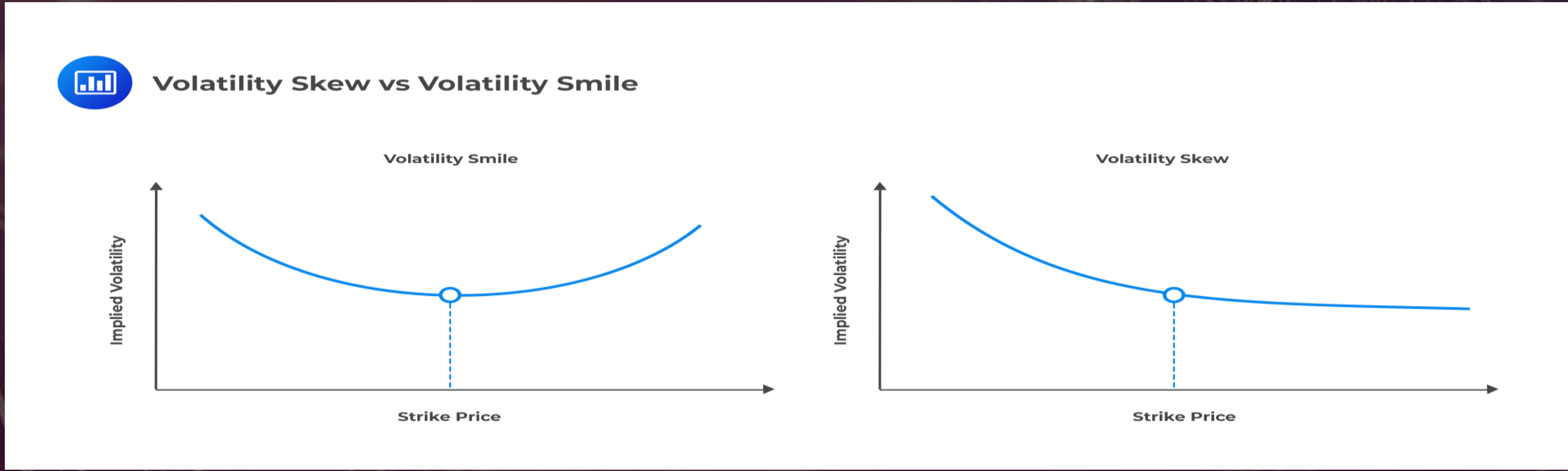
Reality:

- Rates and volatility are not constant
- There are transaction and taxation costs
- Arbitrage opportunities exist (mispricing)





CONCRETE CASE STUDY : TRADING THE VOLATILITY SMILE

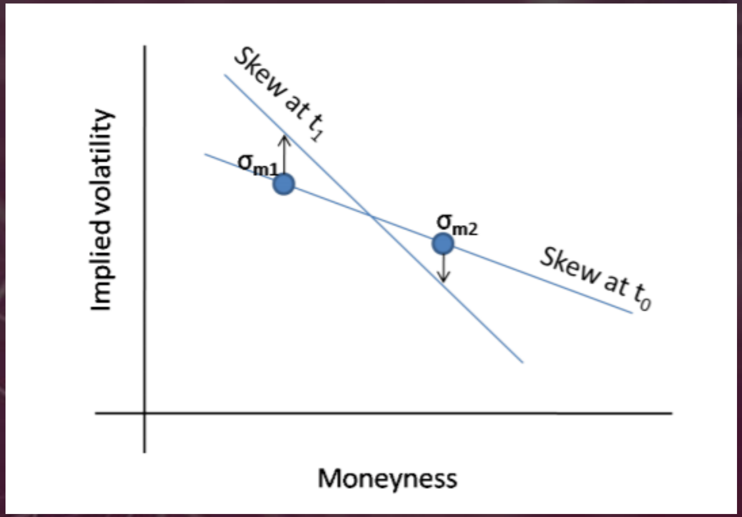


- It defines the structure of implied volatilities for options that have the same expiry and for each strike price.
- The volatilities of itm and otm options are greater than the volatilities of atm.
- The symmetric shape of the smiley face is found in currency options, which have thick tails in positive and negative price movements following the intervention of central banks

- It defines the structure of implied volatilities for options that have the same expiry and for each strike price.
- Volatility decreases as the strike price increases.
- The asymmetric shape of the smile is found in equity options, which have thick left tails in negative price movements (result of relevant news eg: earnings, fed, ecb) and thinner right tail in cases of price rise.

CONCRETE CASE STUDY: TRADING THE VOLATILITY SMILE

SPREAD VOLATILITY TRADE



STEP 1: Define the entry rule-exit rule model, based for example on the mean reversion of the smiley slope.

STEP 2: identify which volatilities to sell and which to buy by taking advantage of a shift in the volatility smile. In this case I sell sigma2 and buy sigma1 (see fig.) based on the shift I expect.

STEP 3: Remember to create delta hedging to exploit only the increases/decreases in volatility of the underlying.

TRADE CAREFULLY!

References:

-Opzioni, Futures and other derivatives. John C. Hull 11 edition. (Chapter 19 and 20)